Airborne sound transmission of a cross-laminated timber plate with orthotropic stiffness

Luboš Krajči
Swiss Federal Laboratories for Materials Science and Technology, Laboratory for Acoustics/Noise Control, CH - Dübendorf, Switzerland

Carl Hopkins
Acoustics Research Unit, School of Architecture, University of Liverpool, United Kingdom.

John Laurence Davy
School of Applied Sciences, RMIT University, GPO Box 2476V Melbourne, Victoria 3001, Australia

Hans-Martin Tröbs
Laboratory for Acoustics, University of Applied Sciences, Rosenheim, Germany

Summary
This paper concerns the airborne sound insulation of a cross laminated timber plate element used as a wall in buildings. The wall comprised three layers. Hence ultrasonic equipment was used to measure the longitudinal wavespeed in each layer. This confirmed that the plate was orthotropic. Comparison of the measured driving-point mobility with infinite plate models indicated that the stiffness in the vertical direction tends to dominate the dynamic behaviour of the plate when the response is multi-modal. Measurements of the radiation efficiency with mechanical and airborne excitation were shown to have reasonable agreement with theory below the critical frequency. The measured sound reduction index showed reasonable agreement with the prediction for a finite isotropic plate using the equivalent bending stiffness below the lowest critical frequency. Above the highest critical frequency the isotropic and orthotropic models are similar and neither model agreed with the measurements.

PACS no. 43.55.Rg, 43.55.Ti, 43.40.Rj, 43.20.Rz

1. Introduction

Typical Swiss timber based wall constructions are assemblies of many material layers coupled using point or line connectors. Solid cross laminated timber (CLT) plates are used as load-bearing wall partitions due to their static properties and their large surface dimensions.

This paper investigates whether these CLT are orthotropic and whether it is possible to use finite or infinite plate theory to predict the airborne sound insulation.

2. Cross laminated timber plate

The tested cross laminated timber (CLT) plate is constructed from three layers of wood measuring 15–50–15 mm x 35 mm (fir or spruce timber) which are glued together at right angles to each other (see Figure 1). Table I shows the plate parameters.

Figure 1. Cross-section of the CLT plate.
Table I. CLT plate dimensions and mass.

<table>
<thead>
<tr>
<th>x [mm]</th>
<th>y [mm]</th>
<th>z [mm]</th>
<th>Weight [kg]</th>
<th>Density [kg/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4180</td>
<td>2890</td>
<td>80</td>
<td>455</td>
<td>438</td>
</tr>
</tbody>
</table>

3. Measurements and calculations

3.1. Longitudinal wave speed, bending stiffness and critical frequency

For orthotropic walls, the critical frequencies are \( f_L \) and \( f_T \) in the x- and y-directions. These define an effective critical frequency \( f_{L,\text{eff}} \) which can be used to model the plate as an equivalent isotropic plate. From [1],

\[
f_{ci} = \frac{c_{L,i}^2 \pi h}{\rho} \quad \text{where} \quad i = x \text{ or } y.
\]  

\[
f_{L,\text{eff}} = \sqrt{f_{cx} f_{cy}}.
\]

\[
B_i = \frac{\rho c_{L,i}^2 h^3}{12} \quad \text{where} \quad i = x \text{ or } y.
\]

\[
B_{L,\text{eff}} = \sqrt{B_x B_y}.
\]

The thickness of the wall is \( h \). The longitudinal wave speeds \( c_{L,i} \) in different directions have been measured using “Silvatest” ultrasonic test equipment. This test method transmits elastic waves into the material being tested by means of a special transducer at an ultrasonic frequency (50-100 kHz). The excitation is then detected by another transducer placed at the opposite end of the material. The device is thus able to measure the time in microseconds taken by the first wave to travel from the emitting probe to the receiving probe. The wave generated by the pulse can be broken down into a longitudinal wave (known as the primary wave) and a crosswise wave (known as the secondary wave). Their respective speeds are defined respectively as the speed of the longitudinal waves and the speed of the transverse waves. The longitudinal waves have displacements in the direction of propagation, whereas crosswise waves have displacements that are perpendicular to the direction of propagation. One feature of ultrasonic longitudinal waves is that if they meet a layer of air, as in a micro-crack, they are almost completely reflected because of the characteristic impedance mismatch for these small wavelengths. It is assumed that the tested element had no cracks inside it. The measured values are shown in Table II.

Figure 2. Directions for longitudinal wavespeed measurements across the entire plate.

Table II. Measured longitudinal wavespeed on the CLT plate and calculated properties.

<table>
<thead>
<tr>
<th>Direction</th>
<th>1L - 1R</th>
<th>4u - 4d</th>
<th>Average of 2L-2R and 3L-3R</th>
<th>Average of 5u-5d and 6u-6d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured ( c_L ) (m/s)</td>
<td>4670</td>
<td>2455</td>
<td>3327</td>
<td>3984</td>
</tr>
<tr>
<td>Calculated ( f_L ) (Hz)</td>
<td>178</td>
<td>338</td>
<td>250</td>
<td>208</td>
</tr>
<tr>
<td>Calculated B (Nm)</td>
<td>407633</td>
<td>112669</td>
<td>206834</td>
<td>296642</td>
</tr>
</tbody>
</table>

Table III. Effective properties of the CLT plate calculated from measured longitudinal wavespeeds.

<table>
<thead>
<tr>
<th>Direction</th>
<th>Centre section</th>
<th>Outer sheet</th>
<th>Outer sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )-direction</td>
<td>1L - 1R</td>
<td>2L - 2R</td>
<td>3L - 3R</td>
</tr>
<tr>
<td>( y )-direction</td>
<td>4u - 4d</td>
<td>5u - 5d</td>
<td>6u - 6d</td>
</tr>
<tr>
<td>Effective ( c_L ) (m/s)</td>
<td>3386</td>
<td>3606</td>
<td>3675</td>
</tr>
<tr>
<td>Effective ( f_L ) (Hz)</td>
<td>245</td>
<td>230</td>
<td>226</td>
</tr>
<tr>
<td>Effective B (Nm)</td>
<td>214308</td>
<td>242998</td>
<td>252442</td>
</tr>
</tbody>
</table>

From equations (1)-(4) the critical frequencies of orthotropic plate and effective longitudinal wave speed \( c_{L,\text{eff}} \) are shown in Table III. To model the plate as an equivalent isotropic plate, a single value is needed; hence the arithmetic average of
the centre and outer sections is calculated as \( c_1 = 3516 \text{m/s} \).

The longitudinal wavespeed measurements confirm that the CLT plate is orthotropic. However, it is different to other sandwich plates because in the \( x \)-direction, the bending stiffness of the core is twice as large as in the outer layer whereas in the \( y \)-direction, the bending stiffness of the core is 0.4 times larger than the outer layer.

3.2. Driving point mobility

The driving-point mobility of the test object has been measured when fixed installed in the frame of horizontal transmission suite and when freely suspended on a crane. Figure 3 compares the driving-point mobility measurements with the infinite plate theory for an equivalent isotropic plate. Between 100Hz and 1kHz where the response is multi-modal, there is better agreement when \( c_L = 2455 \text{m/s} \) which corresponds to the value in the direction \( 4u-4d \) (along the middle core in the vertical direction) rather than the effective value, \( c_1 = 3516 \text{m/s} \).

![Figure 3. Comparison of measured and predicted driving point mobility.](image)

Below 100Hz, further insight into the appropriate bending stiffness to model the CLT as an equivalent isotropic plate is found by comparing predicted mode frequencies for \( c_L = 3516 \text{m/s} \) with the peak responses in the measured driving-point mobility for the two different boundary conditions. As an approximation it is reasonable to assume that when the plate is in the frame the boundaries will be simply supported, but when freely-suspended there is no ambiguity and all boundaries can be modelled as free. In the frame, the first two modes are predicted to occur at 23 and 45Hz, compared with measured peaks at 20 and 30Hz. When freely suspended the first three modes are predicted to occur at 15, 16 and 35Hz, compared with measured peaks at 14, 24 and 35Hz. Hence it is concluded that an effective stiffness is only appropriate for the first few modes, and that at higher frequencies where the plate has a multi-modal response, the \( y \)-direction stiffness dominates.

3.3. Measuring of radiation efficiency

The radiation efficiency was measured with airborne and mechanical excitation with the CLT plate installed in a horizontal test facility at Empa that had suppressed flanking sound transmission. Airborne excitation used a moveable loudspeaker. For mechanical excitation an electro-dynamic shaker was used at two different positions. The sound pressure level \( L_P \) was sampled using a rotating microphone in the receiving room. The velocity level \( L_V \) on the plate was measured simultaneously with an accelerometer. Equation (5) was used to calculate the radiation efficiency for each form of excitation,

\[
10 \log (\sigma) = L_P - 6 - L_s + 10 \log \left( \frac{A}{S} \right)
\]

Note that \( \sigma_0 \) indicates airborne excitation and \( \sigma_s \) indicates structural excitation. The measured radiation efficiencies using both excitation methods are shown in figure 4.

4. Prediction methods

The following sections describe how to calculate the non-resonant radiation efficiency, the resonant radiation efficiency, the airborne diffuse field excited radiation efficiency and the sound reduction index of a single leaf homogeneous isotropic wall. These equations come from [2-5], [6, 7, 8] and [9].

4.1. Prediction of radiation efficiency

Define three empirical constants.

\[
\begin{align*}
 n &= 2 \\
 w &= 1.3 \\
 \beta &= 0.124
\end{align*}
\]

Define the length of the side of an equivalent square panel

\[
2a = \frac{4S}{U}
\]
where \( S \) is the area of the panel and \( U \) is the perimeter of the panel.

Calculate some intermediate values using the wave number
\[
k = \frac{2\pi f}{c}
\]
(10)
where \( c \) is the speed of sound in air and \( f \) is the frequency.

\[
z = \begin{cases} \frac{w}{2ka} & \text{if } w, \frac{\pi}{2ka} \leq 1 \\ 1 & \text{if } w, \frac{\pi}{2ka} > 1 \end{cases}
\]
(11)

\[
h = \frac{1}{2} \sqrt{\frac{2ka}{\pi}} - \beta
\]
(12)

\[
\gamma = \frac{h}{z} - 1
\]
(13)

\[
y = \frac{2\pi}{k^2 S}
\]
(14)

Calculate the non-resonant radiation efficiency.
\[
\sigma_{nr} = \ln \left( \frac{1 + \sqrt{1 + z^2}}{z + \sqrt{z^2 + y^2}} \right) + \frac{1}{\gamma} \ln \left( \frac{h + \sqrt{h^2 + y^2}}{z + \sqrt{z^2 + y^2}} \right)
\]
(15)

Calculate some more intermediate values.
\[
t = \frac{f}{f_c}
\]
(16)

\[
g = \begin{cases} \sqrt{1 - \frac{1}{t}} & \text{if } f \geq f_c \\ 0 & \text{if } f < f_c \end{cases}
\]
(17)

where \( f_c \) is the critical frequency of the panel.

\[
\sigma_2 = \begin{cases} \frac{1}{\sqrt{g^* + y^*}} & \text{if } 1 \geq g \geq z \\ \frac{1}{\sqrt{(h - \gamma g)^* + y^*}} & \text{if } z > g \geq 0 \end{cases}
\]
(18)

If \( f < f_c \), calculate \( \sigma_1 \).

\[
\sigma_1 = \frac{c}{\pi f_c S} \left[ (1-t) \ln \left( \frac{(1+\sqrt{t})}{(1-\sqrt{t})} \right) + 2\sqrt{t} \right].
\]
(19)

Calculate the resonant radiation efficiency.

\[
\sigma_r = \begin{cases} \min(\sigma_1, \sigma_2) & \text{if } f < f_c \\ \sigma_2 & \text{if } f \geq f_c \end{cases}
\]
(20)

Calculate another intermediate value
\[
r = \frac{\pi \sigma_r}{4\eta}
\]
(21)

where \( \eta \) is the total in situ damping loss factor.

Calculate the airborne diffuse field excited radiation efficiency
\[
\sigma_a = \begin{cases} \frac{r \sigma_r + \sigma_{nr}}{r+1} & \text{if } f < f_c \\ \sigma_r & \text{if } f \geq f_c \end{cases}
\]
(22)

and the ratio of the resonant radiation efficiency to the airborne diffuse field excited radiation efficiency
\[
\frac{\sigma_r}{\sigma_a} = \begin{cases} \frac{r+1}{r+\sigma_{nr} / \sigma_r} & \text{if } f < f_c \\ 1 & \text{if } f \geq f_c \end{cases}
\]
(23)

Figure 4. Comparison of measured and predicted radiation efficiencies by airborne and structural excitation.

Figure 4 shows a comparison of the predicted airborne and structural radiation efficiencies using the isotropic theory with one airborne and two structural radiation efficiency measurements. Note that the airborne and structural radiation efficiencies differ below the effective critical frequency of 236 Hz for both airborne and mechanical excitation. Below the lowest critical frequency of the plate there is reasonable agreement between theory and measurement. This
could only be improved in some frequency ranges at the expense of worsening the disagreement in other frequency ranges by varying the effective bending stiffness.

4.2. Sound insulation of a single leaf wall – isotropic plate theory

This section describes how to calculate the sound insulation of single leaf homogeneous isotropic wall using the theory given in [3].

Calculate

\[ A = \frac{\pi f m}{\rho_0 c} \]  

where \( m \) is the mass per unit area of the panel and \( \rho_0 \) is the density of air.

Calculate

\[ B = \frac{\sigma_x}{A} \]  

\[ \tau_1 = \frac{(B)^2}{2(1+\eta)} \left\{ \arctan \left[ \frac{2}{B + \eta} \right] - \arctan \left[ \frac{2(1-\Gamma)}{B + \eta} \right] \right\} \]  

The sound transmission factor is

\[ \tau = \begin{cases} \frac{2\sigma_x r_1}{A^2} + \tau_2 & \text{if } f < f_c, \\ \tau_2 & \text{if } f \geq f_c \end{cases} \]  

The sound transmission factor should be averaged over at least three different frequencies in the frequency band containing the critical frequency. The sound reduction index is then calculated using

\[ R = -10 \log_{10}(\tau) \]  

4.3. Orthotropic plate model

An infinite orthotropic plate model from Heckl [11] and described in [1] is used for comparison with the equivalent isotropic plate model.

Using a similar approach as for an isotropic infinite plate [1], the angle-dependent transmission coefficient for an infinite orthotropic plate with mass and stiffness can be calculated and the diffuse incidence transmission coefficient calculated using the appropriate integration over \( \theta \) and \( \phi \) according to

\[ \tau_{\phi,\theta} = \frac{2}{\pi} \int_{0}^{\pi} \int_{0}^{\frac{\pi}{2}} \frac{d(\sin^2 \theta) d\phi}{1 + \frac{Z_r \cos \theta}{2\rho_0 c_0}} \]  

where the surface impedance in terms of the bending wavenumber in the x and y-directions is given by

\[ Z_r = i\omega \rho_0 \left[ 1 - \left( \frac{\cos^2 \phi + \sin^2 \phi}{k_{B,x}^2} \right)^2 k^4 \sin^4 \theta \right] \]  

The effect of damping is included in the surface impedance by using \( B_{p,x}(1+i\eta) \) and \( B_{p,y}(1+i\eta) \) to calculate the bending wavenumbers.

4.4. Comparison of measured and predicted airborne sound insulation

Figure 5 allows comparison of the measured airborne sound insulation with the finite plate isotropic model using an equivalent bending stiffness, and the infinite plate orthotropic model.

The predictions used the measured total loss factors calculated from the structural reverberation time, \( T_s \).

The orthotropic plate model used the longitudinal wavespeeds measured on the centre section in directions 1L-1R and 4u-4d. The equivalent isotropic plate model uses the effective longitudinal wavespeed, 3386m/s.

The measured sound reduction index has a prominent dip associated with its two critical frequencies in the 250 and 315Hz one-third octave bands. Below this dip the finite isotropic plate model shows reasonable agreement with measurements. Above this dip the isotropic and orthotropic models are similar and neither model shows good agreement with measurements. The reason for this is not yet known and requires further investigation.

The only orthotropic model that is available is for an infinite plate: hence to improve agreement there is scope for a future investigation to use spatial widening to convert the infinite panel results to finite panel results [12].
5. Conclusions

Investigation of the dynamic properties of a cross laminated timber plate using ultrasonic equipment has been used to determine the longitudinal wavespeeds of the inner and outer layers. This confirmed that the plate is orthotropic.

Comparison of the measured driving-point mobility with infinite plate models indicates that the stiffness in the vertical direction tends to dominate the dynamic behaviour of the plate when the response is multi-modal. Reasonable estimates of the local mode frequencies by treating the plate as an equivalent isotropic plate were only obtained for the first few modes below 100Hz.

The radiation efficiency was measured in a transmission suite using both airborne and mechanical excitation. Below the lowest critical frequency of the plate there was reasonable agreement between theory and these measurements.

The measured sound reduction index showed reasonable agreement with the prediction for a finite isotropic plate using the equivalent bending stiffness below the lowest critical frequency. Above the highest critical frequency the isotropic and orthotropic models are similar and neither model shows good agreement with measurements. The reason for this is not yet known and requires further investigation along with further development of orthotropic models for finite size panels to improve its prediction below the lowest critical frequency.

References